



# Effect of Nonzero Surface Admittance on Receptivity and Stability of Compressible Boundary Layer

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## Abstract

The effect of small-amplitude short-scale variations in surface admittance on the acoustic receptivity and stability of two-dimensional compressible boundary layers is examined. In the linearized limit, the two problems are shown to be related both physically and mathematically. This connection between the two problems is used, in conjunction with some previously reported receptivity results, to infer the modification of stability properties due to surface permeability. Numerical calculations are carried out for a self-similar flat-plate boundary layer at subsonic and low supersonic speeds. Variations in mean suction velocity at the perforated admittance surface can also induce receptivity to an acoustic wave. For a subsonic boundary layer, the dependence of admittance-induced receptivity on the acoustic-wave orientation is significantly different from that of the receptivity produced via mean suction variation. The admittance-induced receptivity is generally independent of the angle of acoustic incidence, except in a relatively narrow range of upstream-traveling waves for which the receptivity becomes weaker. However, this range of angles is precisely that for which the suction-induced receptivity tends to be large. At supersonic Mach numbers, the admittance-induced receptivity to slow acoustic modes is relatively weaker than that in the case of the fast acoustic modes. We also find that purely real values for the surface admittance tend to have a destabilizing effect on the evolution of an instability wave over a slightly permeable surface. The limits on the validity of the linearized approximation are also assessed in one specific case.

# 1 Introduction

Reduction of aerodynamic drag through laminar flow control (LFC) usually involves the use of boundary-layer suction through a perforated surface. Suction makes the boundary-layer profiles fuller, so that the resulting boundary-layer flow is more stable than that in the absence of any suction [1]. For structural reasons, the suction velocity distribution is not continuous in practice but is concentrated in the form of discrete suction strips [2], [3], [4]. Strip suction has an effect on stability similar to that of continuous suction and, if optimized, can yield even higher reductions in the growth rates of the instability waves [3].

The nonzero admittance of a perforated suction surface has no effect on the mean boundary-layer flow if the mean pressure on both sides of the surface is assumed to be the same. But, by allowing nonzero normal-velocity fluctuations at the surface, it can still influence the evolution of the instability waves. The effect of surface admittance on boundary-layer stability was first recognized by Gaponov [5]; he made theoretical predictions for the surface admittance produced by a hypothetical design of the suction system and went on to compute the resulting changes in the critical Reynolds number  $R_{\text{crit}}$  (below which no perturbations can amplify on a linear basis) and in the neutral stability curve for small-amplitude perturbations. Although the knowledge of  $R_{\text{crit}}$  and of the neutral stability curve is quite useful, more significant information about the stability of a flow is usually provided by the linear growth rates of the instability waves. Calculations of admittance-induced changes in the growth rates of instability waves were carried out by Lekoudis [6] for the case of a realistic wing geometry. Lekoudis found that significant stabilization of the boundary-layer flow was possible for one specific porous-surface configuration. However, we must note that he used the incompressible form of the linear stability equations to examine the stability of the boundary layer, even though the upstream Mach number was as large as 0.82. Moreover, the surface admittance was calculated using a two-dimensional model for the suction system, which consisted of slits rather than the perforations that might be encountered in practice.

The calculations of both Gaponov and Lekoudis were based on the assumption of a locally uniform surface admittance. A short-scale variation in the surface admittance distribution (which occurs on a streamwise scale that is comparable to or shorter than the wavelength of the instability wave) will lead to a scattering of those instability waves that are generated upstream of the region where this variation takes place. Part of the scattered field will consist of the instability wave, so that the main effect of the

variation will be to modify the amplitude of the instability wave downstream of the variation. Zhang and Kerschen [7] used an asymptotic model to study the scattering process near a perforated strip of moderately large admittance in an otherwise impermeable surface. In particular, the scattering of energy from a TS wave to the strongly damped higher eigenmodes was studied using the triple deck theory. The effect of arbitrary spatial variations in surface admittance was studied using a perturbation expansion for small admittance values.

The other major consequence of short-scale variations in surface admittance is that such variations can scatter free-stream acoustic waves into instability modes [8]. The excitation of instability waves by environmental disturbances is known as the receptivity phase of the laminar-turbulent transition process [9]. Receptivity of boundary-layer flows has been an active area of research during the past decade, mainly as the result of the theoretical breakthroughs made by Goldstein [10], [11]. For a review of the early work on boundary-layer receptivity in low-speed boundary layers, the reader is referred to Goldstein and Hultgren [12]. We note that in the case of boundary-layer flow over a perforated suction surface, receptivity can be produced via variations in both the surface admittance and the mean suction velocity [8], [13]. However, when the amplitudes of both variations are small, the associated receptivity mechanisms are linear and can, therefore, be studied independently.

In this paper, we apply the ideas of refs. [8], [11], and [14] to make quantitative predictions for the receptivity produced by small-amplitude surface-admittance variations in a two-dimensional compressible boundary layer. The results confirm the asymptotic prediction [15] that in low-Mach-number flows the magnitude of admittance-induced receptivity is independent of the orientation of the incident acoustic wave. We also demonstrate how the above isotropic behavior is modified in the range of transonic and supersonic Mach numbers. The related receptivity mechanism that involves variations in mean suction velocity has been studied in a companion paper [16]. Herein, we compare the two receptivity mechanisms and predict when one mechanism will dominate over the other one.

The Green's function that is computed in the context of admittance-induced receptivity is an intrinsic property of the local mean profile. It is also shown to be relevant to the problem of instability-wave scattering (or, for that matter, the stability of the boundary-layer flow over a uniformly permeable surface). We utilize this connection to illustrate the influence of weak surface permeability (with or without short-scale variations) on the evolution of an instability wave. In general, the make up of the surface admittance can have a significant effect on the evolution process. Even though a purely resistive

surface impedance leads to a net energy loss at the surface, its effect is usually more destabilizing than that of a purely reactive surface impedance. The change in the Reynolds-stress behavior because of a nonzero surface admittance is also documented.

## 2 Effect of Variations in Surface Admittance

### 2.1 Receptivity

First, consider the receptivity produced by the interaction between a small-amplitude time-harmonic plane acoustic wave and a local region of nonzero surface admittance in an otherwise impermeable surface. The schematic of the problem is shown in figure 1, which also defines the relevant parameters associated with the two-dimensional mean flow and the three-dimensional acoustic disturbance in the free stream. Throughout this paper, all lengths are nondimensionalized by the local displacement thickness  $\delta^*$  of the mean boundary layer; velocities, by the (local) mean free-stream speed  $U_\infty^*$ ; and pressure, by  $\rho_\infty^* U_\infty^{*2}$ . All other thermodynamic quantities are nondimensionalized by their mean free-stream values, whereas the time  $t$  and the frequency  $\omega$  are scaled with respect to  $\delta^*/U_\infty^*$  and its inverse, respectively. The free-stream Mach number  $U_\infty^*/c_{ac,\infty}^*$  in the local region is denoted by  $M$ .

The amplitude  $\epsilon_{ac}$  of the pressure fluctuation associated with the incident acoustic wave is assumed to be sufficiently small, so that the unsteady motion can be treated as a linear perturbation of the mean boundary-layer flow. The Reynolds number  $R_{\delta^*} \equiv U_\infty^* \delta^*/\nu_\infty^*$  is typically large; therefore, the linear stability of the local mean flow is adequately described by the quasi-parallel stability theory. The local distribution of the surface admittance  $\beta \equiv \beta^* \rho_\infty^* U_\infty^*$  is assumed to have been specified in the form  $\beta(X) = \epsilon_\beta F(X)$ , where we assume that  $\epsilon_\beta \ll 1$  so that a regular perturbation analysis may be carried out. (The LFC suction surfaces are weakly porous, with an open area ratio of  $O(0.004)$  [2]. Therefore, the above approximation is also relevant in practice.) The precise restriction on  $\epsilon_\beta$  depends on the nature of the mean boundary-layer profile and the relative scaling of  $\omega$  with respect to  $R_{\delta^*}$ . For example, in the case of viscous-inviscid-interactive Tollmien-Schlichting (TS) modes (for which  $\omega = O(R_{\delta^*}^{-1/4})$ ), we require the stricter constraint  $\epsilon_\beta \ll R_{\delta^*}^{-1/4}$  in order to permit a linearized analysis of the problem. (Refer to the scalings in refs. [10], [11].) Note that the mean pressure is assumed to be the same on both sides of the perforated skin (i.e., no mean suction is applied at the surface).

In view of the conditions stipulated above, the local unsteady motion can be expanded in the form

$$\mathbf{q} \equiv (u, v, w, p, T) = \epsilon_{ac} \left[ \mathbf{q}_0(Y) \exp(i\alpha_{ac}X) + \epsilon_{\beta} \mathbf{q}_1(X, Y) + O(\epsilon_{\beta}^2) \right] \exp[i(\beta_{ac}Z - \omega t)]. \quad (1)$$

To the leading order, the unsteady motion is not affected by the change in surface admittance. Therefore, the zeroth-order solution  $\mathbf{q}_0(Y)$  corresponds to the acoustic-signature field within the boundary-layer flow over an impermeable surface. In the range of frequencies that is relevant to the receptivity problem, the acoustic wavelength is much smaller than the streamwise length scale of the mean boundary layer. Therefore,  $\mathbf{q}_0(Y)$  satisfies the linear, parallel-flow disturbance equations [17], [18], which also govern the stability of the boundary layer. Of course,  $\mathbf{q}_0(Y)$  is governed by a boundary-value problem, unlike the eigenvalue problem that occurs in linear stability theory. The outer boundary conditions on  $\mathbf{q}_0(Y)$  require that in the free-stream region  $\mathbf{q}_0(Y)$  should asymptote to a superposition of (i) the incident wave and (ii) a specularly reflected wave whose amplitude is determined by the acoustic refraction across the mean boundary layer and by the homogeneous boundary conditions relevant to an impermeable surface.

The  $O(\epsilon_{\beta})$  surface admittance produces the scattered field  $\mathbf{q}_1(X, Y)$  which, again, satisfies the parallel-flow disturbance equations after a Fourier transform ( $X \rightarrow \alpha$ ,  $\mathbf{q}_1 \rightarrow \bar{\mathbf{q}}_1$ ) is taken in the streamwise direction. A complete statement of the disturbance equations for compressible shear flows can be found in refs. [17], [19] and will not be repeated here. The scattered field  $\mathbf{q}_1$  is driven by the  $O(\epsilon_{ac}\epsilon_{\beta})$  fluctuation in the normal velocity at the surface, which is produced by the  $O(\epsilon_{ac})$  pressure fluctuation. Thus,  $\bar{v}_1(\alpha, 0)$  is given by

$$\bar{v}_1(\alpha, 0) = -\bar{F}(\alpha)p_0(0) \quad (2a)$$

In general, the solution for  $\mathbf{q}_1$  would require a numerical solution for  $\bar{\mathbf{q}}_1(\alpha, Y)$ , which would be followed by an integration in the complex  $\alpha$  plane

$$\mathbf{q}_1(X, Y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{\mathbf{q}}_1(\alpha, Y) e^{i\alpha X} d\alpha \quad (2b)$$

to invert the Fourier transform. However, in a receptivity problem, interest is usually limited to the part of  $\mathbf{q}_1$  that corresponds to just one eigensolution (or, at most, a few eigensolutions) of the linear stability problem (viz., the instability wave). The fluctuation  $\mathbf{q}_{1,ins}$  that is associated with each instability mode of interest can be determined by isolating the residue contribution to the inverse Fourier integral from the simple-pole type singularity in  $\bar{\mathbf{q}}_1$  at the wave-number location given by the

corresponding (discrete) eigenvalue  $\alpha = \alpha_{ins}(\omega, \beta_{ac}, R_{\delta^*})$  [11], [14], [20]. Alternatively, the method of adjoint eigenfunctions may be employed [21], [22]. In view of the driving condition (2a),  $\mathbf{q}_{1,ins}$  can be written in the form

$$\mathbf{q}_{1,ins}(X, Y) = \bar{F}(\alpha_w) \Lambda(\omega, \beta_{ac}, R_{\delta^*}, \theta_{ac}^p) \mathbf{E}_{ins}(Y) \exp(i\alpha_{ins}X) H(X) \quad (\alpha_w = \alpha_{ins} - \alpha_{ac}) \quad (3)$$

where  $\theta_{ac}^p \equiv |\pi/2 - \theta_{ac}|$  is the polar angle of acoustic incidence and  $H(X)$  denotes the Heaviside function of its argument. The vector  $\mathbf{E}_{ins}(Y)$  of instability-wave eigenfunctions is assumed to have been normalized in such a way that the eigenfunction that corresponds to the streamwise mass flux has a value of unity at the  $Y$  location where this eigenfunction achieves its maximum magnitude. The parametric dependence of  $\mathbf{E}_{ins}$  on  $\omega$ ,  $\beta_{ac}$ , and  $R_{\delta^*}$  will be suppressed in this paper for brevity of notation. With the above normalization for  $\mathbf{E}_{ins}$ , the product  $C \equiv \epsilon_\beta \bar{F}(\alpha_w) \Lambda$  represents the local coupling coefficient<sup>1</sup> that measures the initial amplitude (i.e., the amplitude at  $X = 0$ ) of the perturbation in the streamwise mass flux in the case of an instability wave that is produced by an incident acoustic wave of unit pressure amplitude. The dependence of  $C$  on the geometry of the local variation is characterized by the spectral amplitude  $\epsilon_\beta \bar{F}(\alpha_w)$  of the surface-admittance distribution because it is the Fourier component  $\bar{F}(\alpha_w)$  that tunes the acoustic wave number  $\alpha_{ac}$  to the instability wave number  $\alpha_{ins}$ . The efficiency function  $\Lambda$  is independent of the geometry and depends only on the frequency  $\omega$  and wave number  $\beta_{ac}$  of the generated instability wave, the location  $R_{\delta^*}$  of the variation in surface admittance, and the angle of incidence  $\theta_{ac}^p$  of the incident wave. Physically,  $\Lambda$  corresponds to the normalized coupling coefficient  $(2\pi)^{1/2} C / \epsilon_\beta$  for a point variation in surface admittance (i.e., for  $F(X) = \delta(X)$ ).

Because of (2a), the effect of acoustic-wave orientation  $\theta_{ac}^p$  on  $\Lambda$  can be separated out by writing

$$\Lambda(\omega, \beta_{ac}, R_{\delta^*}, \theta_{ac}^p) = \tilde{\Lambda}(\omega, \beta_{ac}, R_{\delta^*}) \mathcal{C}_p(\omega, \beta_{ac}, R_{\delta^*}, \theta_{ac}^p) \quad (5a)$$

where the surface pressure coefficient  $\mathcal{C}_p \equiv p_0(0)/p_i(0)$  denotes the zeroth-order pressure fluctuation at the surface, normalized by the pressure amplitude of the incident acoustic wave. Note that the function  $\tilde{\Lambda}$  then corresponds to the efficiency function for the surface-vibrator problem of Gaster [23] (or the Green's function computed by Tam[20] in a related but different context) that is driven by the inhomogeneous boundary condition

$$\bar{v}_1(0) = -\bar{F}(\alpha), \quad (5b)$$

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<sup>1</sup>This term was coined by Tam [20] and was used by Goldstein [11] in his pioneering work.

instead of by (2). Computationally,  $\Lambda$  can be conveniently evaluated by first computing  $\tilde{\Lambda}$  and  $\mathcal{C}_p$  separately. In section 2.2, we show that the function  $\tilde{\Lambda}$  is also relevant to the problem of instability-wave scattering by the local variation in surface admittance. For this problem, the results can be obtained as a by-product of solving the receptivity problem.

## 2.2 Scattering of Instability Wave

Now, consider the problem where an instability wave generated upstream encounters the local region of surface-admittance variation. Because this variation is small in amplitude, a perturbation approach can be employed to examine its effect on the growth of the instability wave. Such an approach has been previously used by a number of researchers to examine the effect of small-amplitude surface disturbances on boundary-layer stability. See, for example, refs. [3], [7], [24]. Our intent here is merely to demonstrate the connection between the receptivity problem examined in section 2.1 with the problem of stability modification and to use this connection to infer the effect of surface permeability on the stability of a compressible boundary layer. Along the same lines as (1), the unsteady motion in the present case can be expanded in the form

$$\mathbf{q} \equiv (u, v, w, p, T) = \epsilon_{ins} \left[ \mathbf{q}_0(Y) \exp(i\alpha_{ins} X) + \epsilon_\beta \mathbf{q}_1(X, Y) + O(\epsilon_\beta^2) \right] \exp[i(\beta_{ins} Z - \omega t)]. \quad (6)$$

where the zeroth-order term  $\mathbf{q}_0$  now corresponds to a continuation of the upstream wave to the local region in the absence of any variation in the surface admittance. Therefore, if we identify  $\epsilon_{ins}$  with the local amplitude of the streamwise mass flux associated with the upstream wave (which is the quantity that is measured by a hot-wire anemometer), then  $\mathbf{q}_0(Y)$  represents the set of normalized eigenfunctions  $\mathbf{E}_{ins}(Y)$  for the instability wave over an impermeable surface. The interaction of  $\mathbf{q}_0(Y)$  with the local region of nonzero admittance produces the scattered field  $\mathbf{q}_1(X, Y)$ . From the discussion related to (2), we may conclude that the Fourier transform  $\bar{\mathbf{q}}_1(\alpha, Y)$  of the scattered field will satisfy the linear stability equations, subject to the inhomogeneous boundary condition

$$\bar{v}_1(\alpha, 0) = -p_{ins}(0) \bar{F}_0(\alpha - \alpha_{ins})$$

The part of  $\mathbf{q}_1$  that corresponds to the instability mode can now be expressed in the form

$$\mathbf{q}_{1, ins} = S(\omega, \beta_{ins}, R_\delta) \mathbf{E}_{ins}(Y) \exp(i\alpha_{ins} X) H(X) \quad (7a)$$

where the scaled scattering coefficient  $S$  is given in terms of the efficiency function  $\tilde{\Lambda}$ , which was obtained in the context of the receptivity problem, by

$$S = \bar{F}(0) \Lambda_s(\omega, \beta_{ins}, R_{\delta^*}) \quad (7b)$$

where

$$\Lambda_s(\omega, \beta_{ins}, R_{\delta^*}) = \tilde{\Lambda}(\omega, \beta_{ins}, R_{\delta^*}) p_{ins}(0) \quad (7b)$$

analogous to (5a). Let us comment on the similarity between the receptivity and the scattering problems. Effectively, the scattering region acts as a secondary source of receptivity. The net amplitude of the instability wave in the region downstream of the surface-admittance variation (i.e., the amplitude of the transmitted instability wave) is determined by the interference between the primary (incident) and the secondary (produced by backscattering) waves. Within the linear limit used here, only the mean component (i.e., the Fourier component with zero wave number) of the local admittance distribution can produce a backscattering of the instability wave into or out of itself. Therefore, the correction to the instability-wave amplitude is determined solely by the (complex) Fourier amplitude  $\bar{F}(0)$ . Note that the secondary wave is not a reflected wave, because any instability that is produced locally will appear only on the downstream side of the scattering region.

In the limit of  $F(X) = F_0$  (= Constant), the scattered field consists purely of the instability wave because the surface-admittance distribution only has the mean component  $\alpha = 0$ . However, note that the above geometry corresponds to a locally uniform distribution of the type that was examined by Gaponov [5] and by Lekoudis [6]. Therefore, the evolution of an instability wave in this case could also have been predicted by considering the Wentzel-Kramers-Brillouin (WKB) expansion for the eigenmode solution

$$\mathbf{q}_{ins} = [\mathbf{E}_{ins,0}(Y) + \epsilon_\beta \mathbf{E}_{ins,1}(Y) + \dots] \exp[i \Theta_{ins,0}(X) + \epsilon_\beta i \Theta_{ins,1}(X) + \dots] \quad (8a)$$

where  $\alpha_{ins,0} \equiv \frac{d\Theta_{ins,0}}{dX}$  is the eigenvalue for an impermeable surface and  $\alpha_{ins,1} \equiv \frac{d\Theta_{ins,1}}{dX}$  corresponds to the eigenvalue correction that accounts for the weak permeability of the perforated surface under consideration. The scattering coefficient  $S = F_0 \Lambda_s$  is then related to the eigenvalue correction  $\alpha_{ins,1}$  by

$$S = i\alpha_{ins,1} \quad (8b)$$

Because of the above connection with the eigenvalue problem for a uniformly permeable surface (which can be solved quite easily even when the surface admittance is not small enough to permit linearization),

we are also in a position to assess the limit on the value of  $\epsilon_\beta$  below which the linearized theory in this paper can be expected to be accurate in predicting the receptivity and/or instability-wave scattering in the case of localized surface-admittance distributions.

We also note that when the short-scale variations in the surface-admittance distribution are not localized but are spread out over a large number of instability wavelengths, the results (3) and (7) can be conveniently generalized by accounting for the slow variation in the efficiency function  $\Lambda$  (or  $\hat{\Lambda}$ ), in the wave number  $\alpha_{ins}$  of the instability wave, and in the amplitude and wave numbers of the acoustic wave. (See refs. [25] and [26].) For a flat-plate boundary layer, where the acoustic-wave properties remain uniform, (3) can be generalized by writing

$$\mathbf{q}_{1,ins} = C(X)\mathbf{E}_{ins}(Y)\exp[i\Theta_{ins}(X)] \quad (9a)$$

where

$$C = \frac{1}{(2\pi)^{1/2}} \int^X F \Lambda \exp[-i\Theta_{ins}(X_s)] dX_s. \quad (9b)$$

Closed-form expressions for (9b) can be obtained for certain special geometries; the reader is referred to refs. [26] and [27] for a description of those solutions. For the scattering problem, (9b) will be replaced by

$$C = \frac{1}{(2\pi)^{1/2}} \int^X F \Lambda_s dX_s. \quad (9c)$$

## 3 Results

### 3.1 Receptivity Prediction

#### 3.1.1 Effect of Mach number

The variation of the coupling coefficient in the range of subsonic Mach numbers was examined in the case of the self-similar adiabatic zero-pressure-gradient boundary layer. To simulate wind-tunnel conditions, the stagnation temperature of the flow was kept constant at 311 K as  $M$  was varied. The local admittance distribution was chosen to be that of a single porous strip with a uniform admittance level across its width. The location of the strip was held fixed at  $R = 955$  (where  $R^2$  denotes the Reynolds number  $R_{\ell^*}$  based on the distance  $\ell^*$  from the leading edge to the region of receptivity); we focused on the excitation of the two-dimensional instability wave that begins to amplify just downstream of the strip location. In other words, the frequency of the acoustic wave was chosen to be that corresponding to the lower branch of the neutral stability curve (i.e.,  $\omega = \omega_b$ ). The width of the porous strip was

chosen to be equal to  $\pi/\alpha_w$  to maximize the value of the geometry factor  $\bar{F}(\alpha_w)$  in (3). To obtain a representative magnitude of the coupling coefficient, the angle of incidence of the free-stream acoustic wave was assumed to be fixed at  $\theta_{ac} = 45^\circ$  as  $M$  was varied. We found that the magnitude of the normalized coupling coefficient  $\bar{F}\Lambda$  does not vary significantly across the entire range of subsonic Mach numbers; it changes from a value of 31 at  $M \rightarrow 0$  to 38 as  $M \rightarrow 1$ . Because the acoustic pressure was normalized by  $\rho_\infty^* U_\infty^{*2}$  to obtain the above result, we conclude that the (nondimensional) amplitude of the generated instability wave will increase roughly as  $1/M^2$  as  $M \rightarrow 0$  if the sound pressure level (SPL) is kept fixed at a given decibel level as the Mach number is decreased.

### 3.1.2 Effect of $\theta_{ac}$

As indicated by (5a), the variation of the efficiency function  $|\Lambda|$  with the acoustic-wave orientation is entirely characterized by the dependence of the surface-pressure coefficient  $C_p$  on  $\theta_{ac}$ . In figure 2, we have plotted the magnitude of  $C_p$  in the case described in the previous paragraph as a function of  $\theta_{ac}$  at a few selected Mach numbers. When the Mach number is small,  $|C_p|$  (and, hence, the magnitude of the coupling coefficient) is independent of the acoustic-wave orientation. Because the boundary-layer thickness is uniformly small as compared with the wavelength of an acoustic wave, the reflected pressure wave is almost in phase with the incident wave; therefore,  $|C_p| \approx 2.0$  at all values of  $\theta_{ac}$  when  $M$  is small. This isotropic property of admittance-induced receptivity is also predicted in ref. [15] on the basis of an asymptotic model based on the triple-deck theory [11], [14], [28]. However, the change in the  $|C_p|$  curve as  $M$  increases beyond 0.5 is even more interesting. The refraction of the acoustic waves across the mean boundary layer introduces a nonzero phase difference between the surface-pressure fluctuations associated with the incident and the reflected acoustic waves. Consequently, the magnitude of  $C_p$  decreases sharply as  $\theta_{ac}$  is increased above some critical value; this value lies in the range of acoustic waves that are incident from the upstream direction.

In the range of supersonic Mach numbers ( $M > 1$ ), the acoustic modes at any fixed frequency become two disjointed sets of modes, namely the fast mode that is analogous to acoustic waves in subsonic flows and the slow mode that has a narrower range of possible orientations and, moreover, has a critical layer within the mean boundary-layer region. Overall, the slow-mode waves correspond to smaller values of  $|C_p|$  and, hence, are expected to produce lower receptivity than the fast-mode acoustic disturbances for a given SPL. At a given set of values for  $M$ ,  $R$ , and  $\theta_{ac}$ , the  $|C_p|$  values for the slow

acoustic mode decrease with an increase in the frequency parameter  $\omega$  (i.e., as the acoustic wavelength shortens with respect to the thickness of the mean boundary layer). Moreover, for a given  $\omega$  in the vicinity of the lower branch frequency, the  $|C_p|$  value increases as the Mach number increases.

### 3.1.3 Comparison with Suction-Induced Receptivity

An interesting comparison is made between the coupling coefficients found above for the admittance-induced receptivity and those obtained in ref. [16] for the related mechanism of receptivity caused by an identical variation in the mean suction velocity. First, the  $\theta_{ac}$  dependence of the  $\Lambda$  function in figure 2 is completely different from that of the efficiency function  $\Lambda^{(s)}$  for the suction-induced receptivity. The latter function has a peak in the range of upstream-traveling incident waves ( $\pi/2 < \theta_{ac} < \pi$ ), and the magnitude of this peak increases as the Mach number increases. In contrast, the  $|\Lambda|$  values for surface-admittance-induced receptivity tend to decrease in the range of  $\theta_{ac}$  where the peak of  $|\Lambda^{(s)}|$  occurs. The decrease in  $|\Lambda|$  is particularly prominent at the higher values of  $M$ . If we compare the maxima of the two efficiency functions at a given Mach number, then to produce the same magnitude of coupling coefficient in both cases, the ratio  $\frac{\epsilon_\beta/M}{V_w^*/U_\infty^*}$  † must be approximately 12 at both  $M = 0.1$  and  $M = 0.9$ . The resistive impedance of a perforated surface is very sensitive to the radius of the pores, and the reactive impedance is determined mostly by the specific configuration of the ducting system underneath the surface. Therefore, we will not draw any conclusions in regard to which receptivity mechanism is likely to dominate in practice. However, the suction velocities required to stabilize a boundary-layer flow are very small ( $V_w^*/U_\infty^* = O(10^{-3})$  to  $O(10^{-4})$ ); hence, the above ratios could easily be realized (and perhaps exceeded) in a practical LFC configuration.

### 3.1.4 Effect of Three-Dimensional Acoustic Waves

To understand the effect of the three dimensionality of the acoustic wave on  $C_p$ , we have plotted  $|C_p|$  in figure 3 as a function of the azimuthal orientation  $\phi_{ac} = \arctan(\beta_{ac}/\alpha_{ac})$  at a fixed  $\omega$  (which is the same as that in figure 2) and a fixed  $\theta_{ac}^p$ . Figure 3 shows that the larger the polar angle of acoustic incidence  $\theta_{ac}^p$ , the more sensitive the value of  $|C_p|$  to the azimuthal orientation  $\phi_{ac}$  in the range  $\phi_{ac} > \pi/2$ . The spanwise wave number of the generated instability wave changes with  $\phi_{ac}$ . However, because the variation in  $\phi_{ins}$  in this case is relatively small (it varies from  $[0^\circ, 1.5^\circ]$  at  $\theta_{ac}^p = 5^\circ$  to  $[0^\circ, 33^\circ]$  at

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† The parameter  $\epsilon_\beta/M$  corresponds to the nondimensional amplitude of  $\beta^* \rho_\infty^* c_{ac, \infty}^*$ , which is the more conventional nondimensionalization for specifying the surface admittance.

$\theta_{ac}^p = 85^\circ$ ), the  $\tilde{\Lambda}$  values do not change significantly across the entire range of  $\phi_{ac}$ . Therefore, the  $|\mathcal{C}_p|$  variation in figure 3 is also roughly analogous to the corresponding variation in the  $|\Lambda|$  function.

### 3.2 Instability-wave Scattering and Stability Modification

Next, we consider the effect of admittance variation on the scattering of instability waves. The surface admittance  $\beta$  is a complex quantity. Therefore, if we vary the argument of  $\beta$  while its magnitude is kept fixed, then the interference process between the primary and the secondary waves is modified. This modification will in turn change the downstream amplitude of the instability wave. Equations (6) and (7) show that for a given  $|\beta|$ , the downstream amplitude of the instability wave will be maximum (in the linear limit) when  $\arg(\tilde{F}(0)) = -\arg(\Lambda_s)$ . This maximum amplitude is  $1 + \epsilon_\beta |\tilde{F}(0)\Lambda_s|$  times larger than the amplitude over an impermeable surface at the same location. The variation of  $|\Lambda_s|$  in the  $\omega$ - $R_{\delta^*}$  space was illustrated in [13] in the context of admittance-induced receptivity in low-Mach-number flows. (See figures 6a through 6c of ref. [13].) We found that for the two-dimensional flat-plate boundary layer,  $\arg(\Lambda_s)$  is usually positive and, moreover, is smaller than  $\pi/4$ . This implies that a more destabilizing effect is produced by surface impedances that are resistance dominated ( $\beta$  nearly real, positive) than by surface impedances that are reactance dominated ( $\beta$  nearly imaginary).

Recall from (7) and (8) that the problems of admittance-induced receptivity and admittance-induced instability-wave scattering are mathematically identical to within one scaling factor. Moreover, when  $F(X) = F_0$  (=Constant), the latter problem reduces to computing the eigenvalue correction  $\alpha_{ins,1}$  that accounts for the weak surface permeability in the case of a quasi-uniform admittance distribution. In figure 4, we have plotted the scaled growth-rate correction  $-\text{Im}(\alpha_{ins,1})/F_0$  as a function of  $R$  for an instability wave that corresponds to a nondimensional frequency parameter of  $f \equiv \omega/R_{\delta^*} = 25E - 6$ . (This frequency is close to the value that is deemed to produce transition in the boundary layer over an impermeable surface according to the  $e^9$  criterion [17].) The Mach number is taken to be equal to 0.02. When the surface is impermeable, the lower and upper branches of the neutral stability curve under the above conditions correspond to  $R \approx 910$  and  $R \approx 1975$ , respectively. For comparison, the predictions for  $\text{Im}(\alpha_{ins} - \alpha_{ins,0})/\epsilon_\beta F_0$  that were obtained from the numerical solution of the nonlinear eigenvalue problem (for sufficiently large values of  $\epsilon_\beta$ ) with nonzero (and real)  $\beta$  are also shown in figure 4. We observe that the linearized result obtained from the perturbation expansion (8a) is useful for surface-admittance values that correspond to approximately  $\epsilon_\beta F_0 < 0.10$ . The deviation from the linear prediction is relatively larger at the lower Reynolds numbers, i.e., at positions upstream of the

lower branch. Even before the linear approximation loses its validity, the predicted growth rates become substantially larger than those over an impermeable surface, so that ignoring the nonzero surface admittance in those cases would lead to an overprediction of the transition Reynolds number according to the  $\epsilon^9$  criterion for transition prediction [17]. The estimates obtained in ref. [29] suggest that the values of surface admittance in a typical LFC application are likely to be well within the linear range. But experimental measurements of the surface admittance must be carried out so that the effect of surface permeability on the N factor may be assessed more accurately.

The effect of a nonzero surface admittance on the flow stability appears to be manifested through a rapid increase with  $Y$  in the magnitude of the Reynolds stress  $\tau$  near the surface (where the mean shear is maximum, so that the most energy can be extracted from the mean flow by a fixed increment in Reynolds stress). Figure 5 shows the Reynolds-stress profile for the case where the surface admittance is purely real. For  $\beta \equiv 0$ , the increase in  $|\tau|$  with  $Y$  is weak, because  $|\tau|$  is proportional to  $Y^3$  as  $Y \rightarrow 0$ ; in contrast, a linear increase with  $Y$  is obtained when  $\beta \neq 0$ . Away from the surface, the Reynolds-stress distribution tends to be qualitatively similar in both cases.

Before we conclude, we will comment on the qualitative changes that may be expected when the surface-admittance becomes moderately large (so that the perturbation expansion (6) is no longer valid). An important class of flows in this respect corresponds to viscous-inviscid interactive disturbances in two-dimensional supersonic boundary layers, for which the linear theory ceases to be valid when  $\epsilon_\beta = O(R_\delta^{-1/4})$  as previously mentioned in section 2. The first subclass of disturbance motions under the interactive label is that of unsteady supersonic disturbances (i.e., disturbance modes that are inclined at angles of smaller than  $\arccos(1/M)$  with respect to the mean-flow direction). The triple-deck theory (which is a large-Reynolds-number asymptotic approximation to the Navier-Stokes equations) predicts that supersonic eigenmodes are completely stable when  $\beta = 0$  (i.e., are always damped). However, we will be interested to determine whether amplifying supersonic instabilities can exist when the admittance is allowed to be nonzero. Such an investigation is important because the values of surface admittance that are required to produce an  $O(1)$  impact on the stability properties are asymptotically small. The second subclass of disturbance motions corresponds to steady disturbances that produce an upstream influence in supersonic boundary layers, such as the interaction of a boundary layer with an incident shock [30]. Passive surface porosity has been suggested in the literature as a technique for mean-flow modification over aerodynamic components. In this context, the nonzero admittance values

can have an especially significant impact on the extent of the upstream influence in the above subclass of problems. In particular, preliminary calculations by the author indicate that the decay rate of the disturbances that produce an upstream influence increases rapidly with the (real) amplitude of the surface-admittance distribution. This increase will tend to inhibit the upstream interaction.

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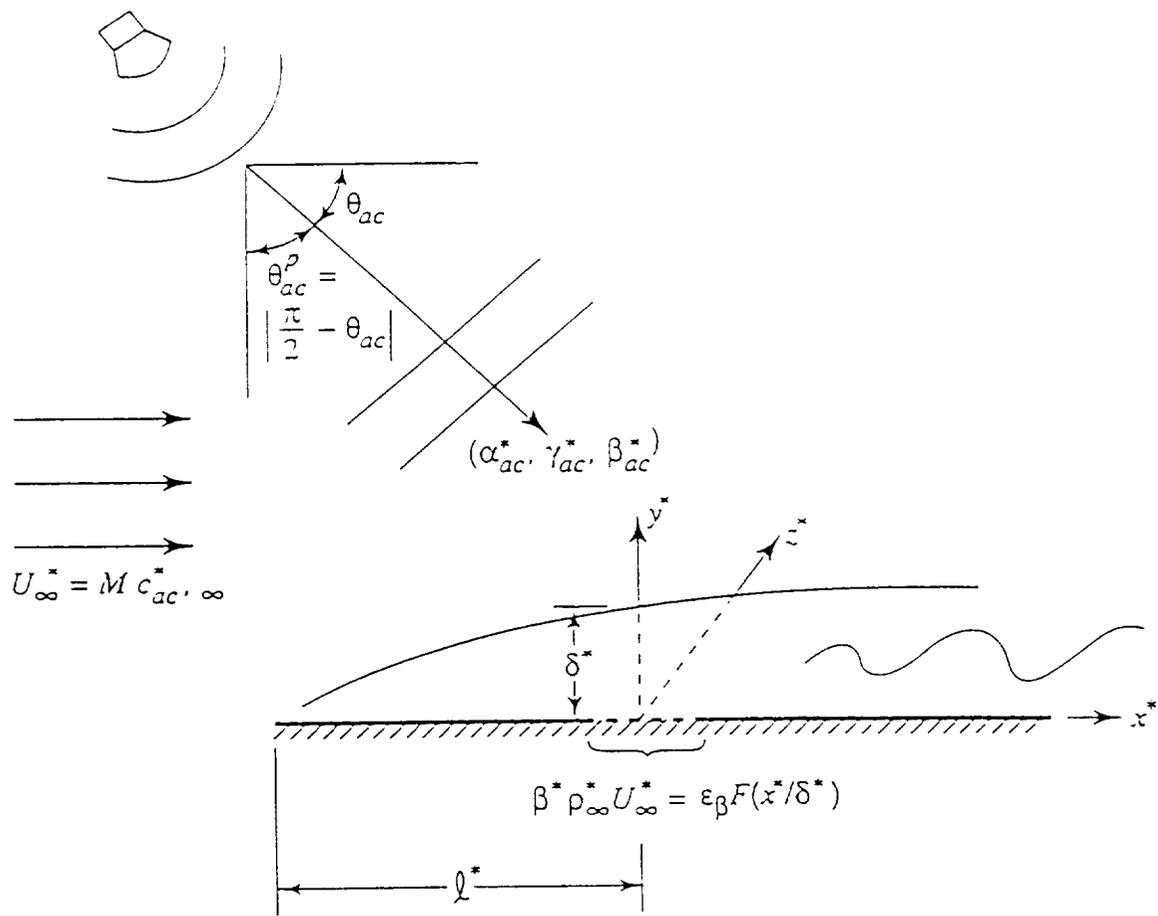


Figure 1. Schematic of problem.

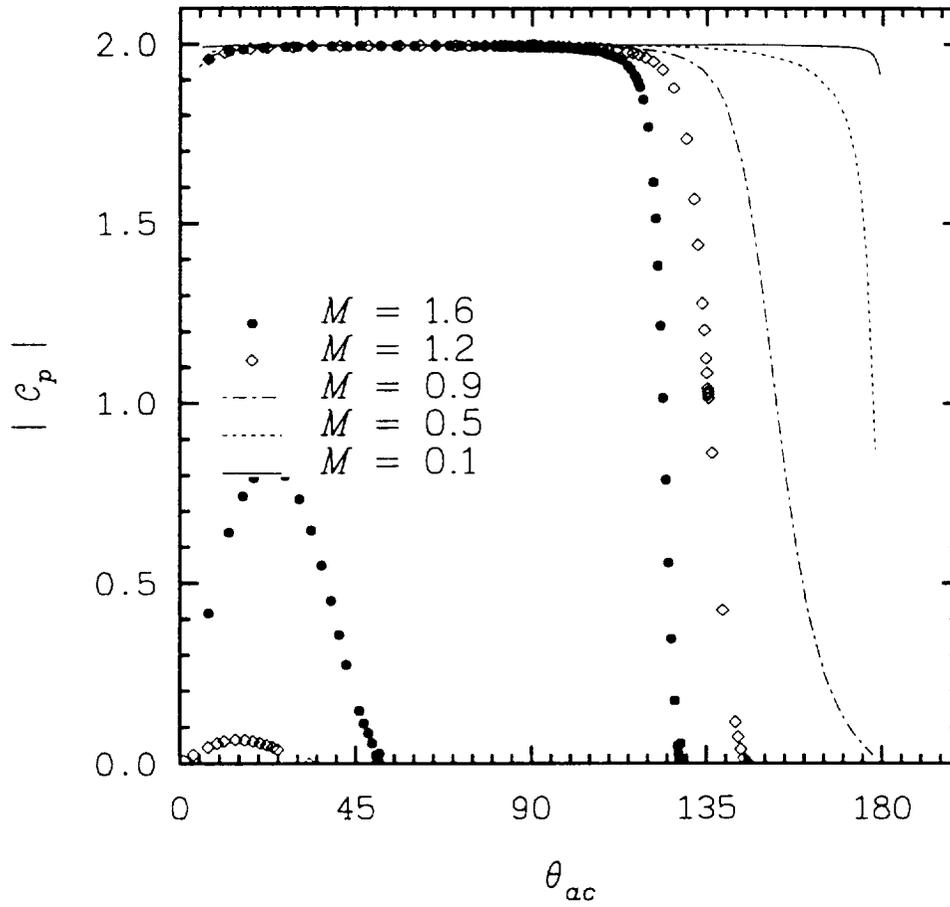


Figure 2. Surface pressure coefficient  $C_p$  as function of  $\theta_{ac}$  for fixed  $\omega$  at selected values of Mach numbers. (For  $T_w/T_{ad} = 1$ ,  $T_{stgn} = 311$  K, and  $\omega = \omega_{lb}$  at  $R = 955$ .)

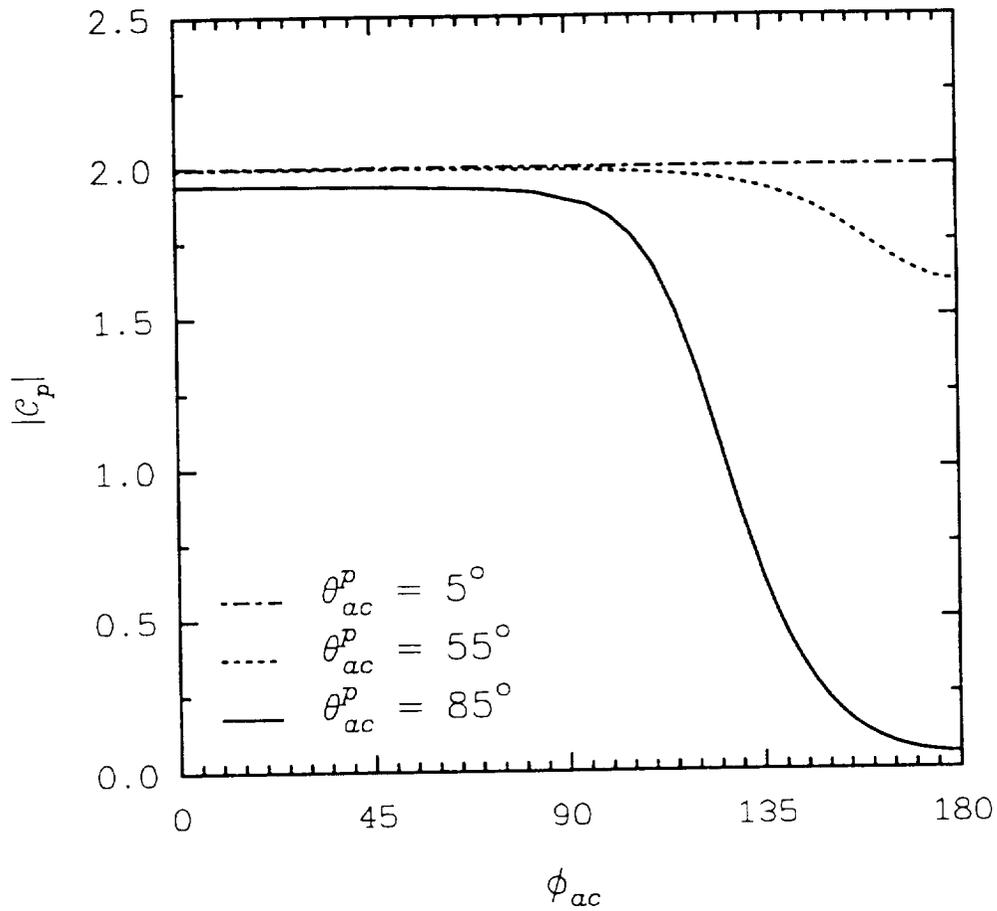


Figure 3. Surface pressure coefficient  $|C_p|$  as function of  $\phi_{ac}$  for fixed  $\omega$  at selected values of  $\theta_{ac}^p$ . ( $M = 0.9$ ,  $T_w/T_{ad} = 1$ ,  $T_{stgn} = 311$  K, and  $\omega = \omega_{lb}$  at  $R=955$ .)

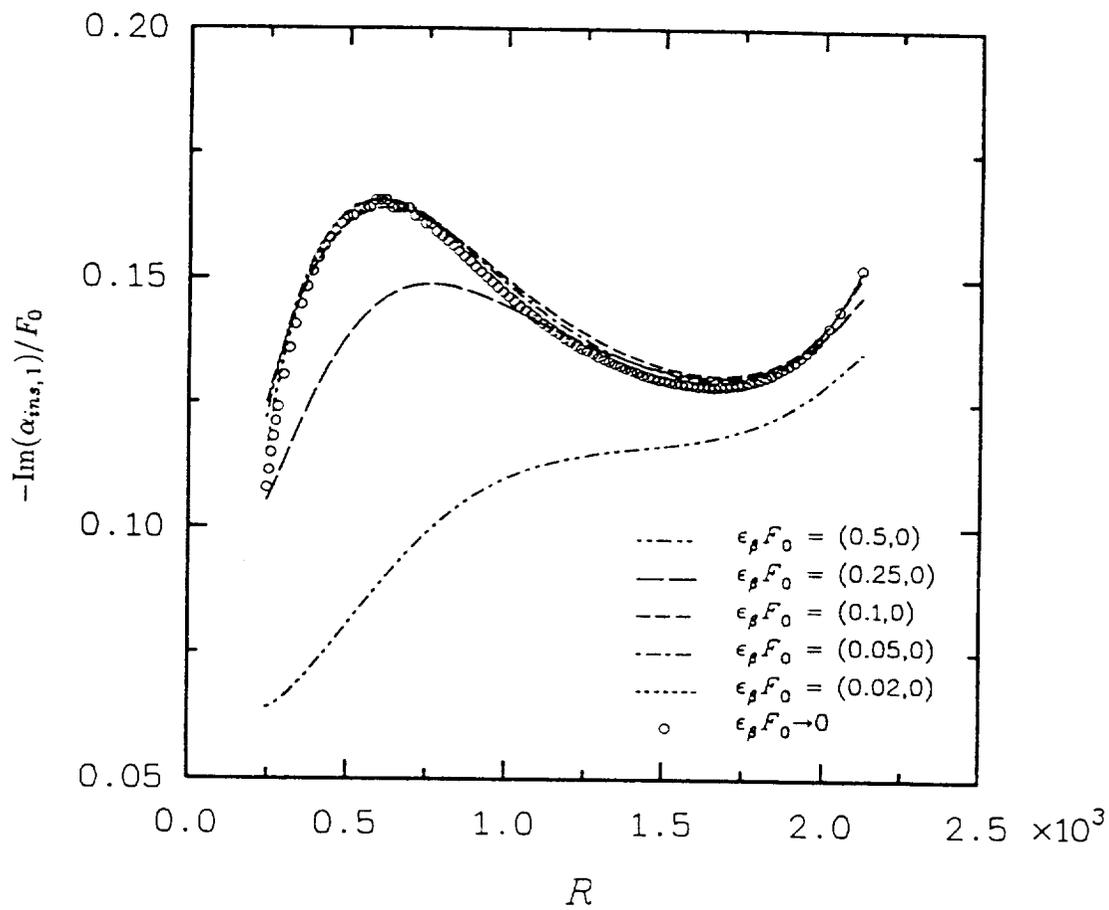


Figure 4. Influence of nonzero (real) surface admittance on boundary-layer stability at  $M = 0.02$  and  $f = 25E-06$ .

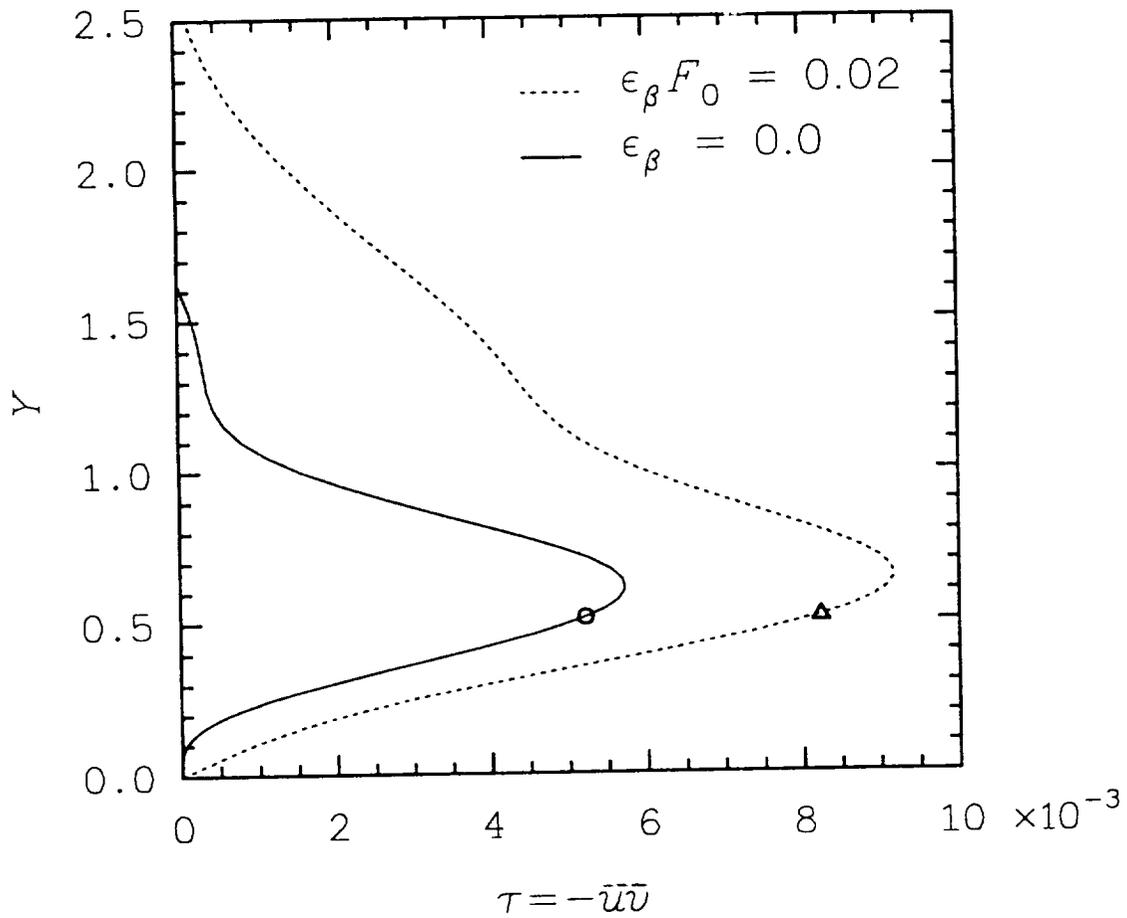


Figure 5. Influence of nonzero (real) surface admittance on Reynolds-stress profile for  $f = \omega/R_\delta \approx 25.0E-06$ ,  $R \approx 920$  and  $M = 0.02$ .

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